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Temporal analysis of math proficiency and factors that impact school performance: an investigation using data from basic education

Análisis temporal del dominio de las matemáticas y los factores que influyen en el rendimiento escolar: una investigación con datos de educación primaria

Analyse temporelle des compétences en mathématiques et des facteurs influençant les performances scolaires : une enquête sur les données de l'enseignement primaire

Análise temporal da proficiência em matemática e fatores que impactam o desempenho escolar: uma investigação com dados da educação básica

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Abstract

During basic education, factors related to family environment and school infrastructure can influence a student's academic performance. Large-scale assessment systems seek to identify mechanisms to improve the quality of education in an effective and efficient manner. Given this context, this study aims to investigate the relationship between mathematics proficiency among third-year high school students from public schools in the state of São Paulo and the schools' pedagogical characteristics and socioeconomic profiles. To this end, data were

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collected on math scores, along with responses from a questionnaire administered to parents of students participating in the 2013 edition of SARESP. The methodology employs two-level hierarchical models, accounting for students nested within schools. The best-fitting model was selected based on the Akaike Information Criterion (AIC), and the analysis was conducted using RStudio. The results indicate that parental education level, family income, and student adherence to homework assignments positively affect mathematics performance. Finally, we would like to point out that failing grades and a lack of teachers for some subjects are detrimental to student learning.

Keywords: Academic performance, Hierarchical generalized linear model, School administration, SARESP.

Resumen

Durante la educación básica, factores relacionados con el entorno familiar y la infraestructura escolar pueden influir en el rendimiento académico del alumno. Los sistemas de evaluación a gran escala buscan identificar mecanismos para mejorar la calidad de la educación de manera eficaz y eficiente. En este contexto, el objetivo de este estudio es investigar la relación entre la competencia en matemáticas de alumnos de tercer año de enseñanza media de escuelas públicas del estado de São Paulo y las características pedagógicas y el perfil socioeconómico de las escuelas. Para ello, se recogieron datos sobre las puntuaciones en matemáticas, junto con las respuestas de un cuestionario administrado a los padres de los estudiantes que participaron en la edición 2013 del SARESP. La metodología emplea modelos jerárquicos de dos niveles, teniendo en cuenta los estudiantes anidados dentro de las escuelas. Se seleccionó el modelo que mejor se ajustaba basándose en el criterio de información de Akaike (AIC), y el análisis se llevó a cabo utilizando RStudio. Los resultados indican que el nivel educativo de los padres, los ingresos familiares y el cumplimiento de los deberes por parte de los alumnos afectan positivamente al rendimiento en matemáticas. Por último, queremos subrayar que los fracasos y la falta de profesores para algunas asignaturas perjudican el aprendizaje de los alumnos.

Palabras-clave: Rendimiento académico, Modelo lineal generalizado jerárquico, Administración escolar, SARESP.

Résumé

Au cours de l'éducation de base, des facteurs liés à l'environnement familial et à l'infrastructure scolaire peuvent influencer les performances académiques d'un élève. Les systèmes d'évaluation à grande échelle cherchent à identifier les mécanismes permettant d'améliorer la qualité de l'éducation de manière efficace et efficiente. Dans ce contexte, cette étude vise à examiner la relation entre les compétences en mathématiques des élèves de troisième année du secondaire des écoles publiques de l'État de São Paulo et les caractéristiques pédagogiques et les profils socio-économiques des écoles. À cette fin, des données ont été collectées sur les résultats en mathématiques, ainsi que des réponses à un questionnaire administré aux parents des élèves participant à l'édition 2013 du SARESP. La méthodologie utilise des modèles hiérarchiques à deux niveaux, prenant en compte les élèves imbriqués dans les écoles. Le modèle le mieux adapté a été sélectionné sur la base du critère d'information d'Akaike (AIC), et l'analyse a été réalisée à l'aide de RStudio. Les résultats indiquent que le niveau d'éducation des parents, le revenu familial et le respect des devoirs par l'élève ont une incidence positive sur les performances en mathématiques. Enfin, nous tenons à souligner que les échecs et le manque d'enseignants dans certaines matières sont préjudiciables à l'apprentissage des élèves.

Mots-clés : Administration scolaire, Administration scolaire, modèle linéaire hiérarchique généralisé, performance scolaire, SARESP.

Resumo

Durante o processo de escolarização básica, fatores relacionados ao ambiente familiar e à infraestrutura da escola podem influenciar o desempenho educacional do indivíduo. Os sistemas de avaliação de larga escala têm a finalidade de encontrar mecanismos para melhorar a qualidade do ensino ofertado à sociedade de forma eficaz e eficiente. Neste sentido, o presente estudo tem como objetivo investigar a relação entre a proficiência em Matemática dos alunos da terceira série do Ensino Médio, matriculados nas escolas públicas do Estado de São Paulo e as características técnico-pedagógicas e do perfil socioeconômico no qual está inserido. Para tanto, foram coletados dados relativos às notas de Matemática, bem como as respostas dadas ao questionário aplicado aos pais dos alunos participantes da edição de 2013 do SARESP. A metodologia contempla a utilização de modelos hierárquicos em dois níveis, considerando alunos aninhados em escolas. A seleção do modelo mais adequado aos dados deu-se de acordo com o Akaike Information Criteron (AIC) e o *software* RStudio foi empregado na modelagem. As estimações obtidas mostram que o nível de escolaridade dos pais, a renda familiar e o cumprimento das tarefas de casa pelo aluno atuam positivamente sobre o desempenho em Matemática. Por fim, destacamos que reprovações e falta de professores para algumas disciplinas geram prejuízos para o aprendizado do aluno.

Palavras-chave*:* Desempenho escolar, Modelo linear generalizado hierárquico, Diretoria de ensino, SARESP.

Temporal analysis of math proficiency and factors that impact school performance: an investigation using data from basic education

According to Article 205 of Brazil's Federal Constitution (FC), promulgated in 1988 and still in force today, education is the right of all citizens. Over time, Constitutional Amendments have been inserted to establish the compulsory age range for students and guarantee access to and permanence in school. At the same time, the state and the family (or legal guardians) have assumed the duty to ensure that individuals are in school at the appropriate age, and are liable to prosecution if this obligation is not met. Still in the educational sphere, Article 206(VII) of the FC establishes the requirement to guarantee a quality standard for the education offered. For this quality to be effectively guaranteed, it first became essential to measure the education provided (Brasil, 1988).

In response to this demand, in 1996 the Guidelines and Bases Law (LDB) was established (Brazil, 1996), which states, in Article 9, item VI, that it is the Union's duty to ensure a national process for evaluating school performance in primary, secondary and higher education, with the aim of establishing priorities and promoting the improvement of teaching quality. It is therefore the Union's official duty to promote external evaluation processes, with the aim of inferring and requiring teaching quality. In light of this, we can understand the initial process and the demand for public policies aimed at education.

Educational evaluation policies aim to assess the quality of education and play an important role in monitoring and developing the educational field. As Souza (2016) argues,

> Evaluation, by presenting its products, expresses an important face of the educational system, but this face cannot be confused with the full results of all the efforts of countless teachers and students (and extensively: principals, pedagogues, non-teaching workers, families, heads of education systems) in the country. Between the need to know and monitor the progress of work in Brazilian schools and, on the other hand, to indicate the goals to be pursued by education and educators (an effect that also arises because of the current evaluation system), educational policy needs to find the right term to have the most accurate information possible about the processes and products of school work and, at the same time, help build more democratic directions for the educational future. (Souza, 2016, pp. 19-20).

According to Minhoto (2016), the assessment process should involve the student's entire living environment, considering family, social, cultural and school characteristics, and not just the grades achieved in a test. According to the author,

> Assessment can be defined as the complex web of relationships established between different constructs. In this sense, exams and tests that measure student proficiency only really become educational assessment when their results are related to other constructs,

such as the students' socioeconomic characteristics; school experiences; motivations for the course; the educational environment; the infrastructure conditions of the educational institution, among others. (Minhoto, 2016, pp. 151).

Assessment models are on the agenda of several studies in the area (Souza, 2016; Minhoto, 2016; Alavarse et al., 2013, Hojas, 2017). Educators and researchers have been debating the methods traditionally used to assess students, especially when the assessments are external. According to Alavarse et al. (2013, pp. 16), "[...] Brazilian basic education became the object of external evaluations, initially presented as necessary for monitoring the performance of its students in standardized tests, which could allow comparisons between networks and schools".

External evaluations, such as the National Basic Education Evaluation System (SAEB), the São Paulo State School Performance Evaluation System (SARESP) and the National High School Exam (ENEM), among others, seek to present indicators in line with the educational reality. To do this, they administer socio-economic questionnaires, not only to students and teachers, but also to parents. These questionnaires play an important role in the assessment, as they help to identify the family and social context in which the student is inserted. In addition, using this instrument, it is possible to develop studies in which correlations between variables and grades achieved are established and investigated.

This article aims to contribute to the debate by applying multilevel modeling to the results of the School Performance Assessment System of the State of São Paulo (SARESP) and thus collaborate with an analysis in which diverse factors are observed. It is hoped that the results obtained can act as tools for the development and elaboration of public educational policies prioritizing the teaching and learning of mathematics in the basic schooling process.

Large-scale assessment: the São Paulo State School Performance Assessment System (SARESP)

In their investigation into the emergence of large-scale assessments in Brazil, Brito and Conceição (2024) define the 1980s as the starting point for discussions on educational assessment. According to the authors, these external assessments were implemented with the aim of evaluating, diagnosing and promoting advances in educational policies, the central objective of which is to improve the quality of education.

Since the 1980s and 1990s, when the need for these assessments began to be debated, countless tests have been created, modified, adapted, readjusted and continue to be constantly reformulated, because education is not static and they must keep up with the scenario, with the duty of bringing society the picture closest to the reality of the present in which they are applied.

In 2024, Brazil's current large-scale assessments include the Basic Education Assessment System (SAEB), the National High School Exam (ENEM) and the National Student Performance Exam (ENADE), each of which is aimed at different specific audiences.

When education is considered a state product, evaluation becomes essential as an instrument for distributing budgetary resources, investing public funds and as a means of measuring the quality of the service offered. At state level, each unit of the federation has the autonomy to define and carry out its own large-scale assessments and educational indices. In the state of São Paulo, on March 29, 1996, the Secretary of Education, by means of SE Resolution No. 27, instituted the São Paulo State School Performance Assessment System (SARESP). According to the document, it was deemed necessary to create a policy for evaluating teaching at state level, in order to be able to coordinate with the National System for Evaluating Basic Education (SAEB/MEC). Among other functions, the Resolution also states that it is essential to recover the quality of teaching in the State of São Paulo, as well as the need to have a tool capable of providing data to support decision-making by the Teaching Departments and/or School Units (SÃO PAULO, 1996).

Currently, the SARESP is mandatory for students in the 2nd, 5th, and 9th grades of elementary school and the 3rd grade of high school. The subjects covered in all tests, since the first edition, are Portuguese Language and Mathematics; however, in some years, other subjects are included to complete the assessment. In addition to the data from the test results, this assessment also generates important information that structures the São Paulo Education Development Index (IDESP), such as school flow, through student pass and fail rates, and school dropout rates (SÃO PAULO, 2016).

Performance assessment in Mathematics, and in other subjects, is supported by the São Paulo State Curriculum. Based on the learning expectations regarding content, competencies, and skills established for each year/grade, SARESP scores are grouped into four levels: Below Basic (AB), Basic (BA), Adequate (AD), and Advanced (AV).

Table 1 contains information regarding proficiency levels, score ranges and descriptions used by SARESP.

Table 1.

Mathematics proficiency levels established by SARESP (Brasil, 2014, p. 6)

Proficiency level	Score range	Rating	Description
Below Basic (AB)	Less than 275	Insufficient	Students demonstrate insufficient mastery of the content, competencies and skills desirable for the year/grade they are in.
Basic (BA)	Equal to or above 275 and less than 350	Sufficient	Students demonstrate minimal mastery of content, competencies and skills, but have the necessary structures to interact with the curriculum proposal in the subsequent year/grade.
Adequate (AD)	Equal to or above 350 and less than 400		Students demonstrate full mastery of the content, competencies and skills desirable for the year/grade they are in.
Advanced (AV)	Equal or above 400	Advanced	Students demonstrate knowledge and mastery of content, skills and abilities above those required in the year/grade they are in.

SARESP involves students, parents, schools, principals, teachers of various subjects, administrators and inspectors, and uses two assessment instruments, the first characterized by the application of tests on two days, and the second corresponding to the application of questionnaires directed at parents and students, principals, coordinating teachers and other teachers (Brasil, 2022).

The questionnaires make it possible to collect information on students' social, economic, cultural and family backgrounds, their school career, study habits and perceptions and expectations of teachers, teaching practice and school management. The questionnaires aimed at school management provide information on academic training and experience, management studies and perceptions of the functioning and conditions of the school.

The application of SARESP results in different products, including report cards and performance reports, technical and pedagogical reports, which are intended for specific purposes that include investigating whether or not previous results have evolved, locating evidence of improvement and teaching weaknesses, among others. Making the results of this assessment available to its participants enables educators to discuss and propose the actions and strategies needed to achieve the targets set for state public education (Brasil, 2014).

Identifying factors associated with school performance using hierarchical models

In educational evaluation, student performance is an important indicator of the effectiveness of the education system. However, this parameter should not be looked at in isolation. Ferrão et al. (2001), Alves and Soares (2008), Laros and Marciano (2008), among other authors, emphasize that analyses of school performance should combine information from the individual's school and social context. This is because there is empirical evidence that students' school performance is influenced not only by their inherent abilities, but also by factors that include the social, economic and cultural elements of their families. More comprehensive aspects are therefore necessary in order to carry out educational evaluations - since any social inequalities have been proven to influence the results of education systems, as noted in the studies by Jesus and Laros (2004) and Laros et al. (2010).

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In this way, the results obtained through educational assessment systems in recent years allow us to validate the results verified in the Brazilian educational area with greater certainty. This is because there have been advances both in methodological resources and, above all, in the current assessment criteria of the National Basic Education Assessment System (SAEB), the São Paulo State School Performance Assessment System (SARESP) and the National High School Exam (ENEM). These systems have therefore provided a secure critical analysis of school performance and achievement in formal education in Brazil.

What is predominant in the evaluations of these systems is the presence of diverse factors, whether of a human, social or cultural nature, which can influence students' school performance (Ferrão et al., 2001; Soares and Alves, 2008; Andrade and Laros, 2007; Bassetto, 2019). Thus, in order to properly identify the variables associated with school performance, it is necessary to consider the educational structures in which individuals are inserted. These should include aspects of the family environment, the availability and quality of school infrastructure and the teaching staff. In this broader context, the appropriate statistical approach for analysis is hierarchical regression.

Hierarchical or multilevel modeling has been widely used in Brazil since the late 1990s to interpret and analyze data from large-scale educational assessments - as evidenced by studies by Ferrão et al. (2001), Soares and Alves (2008), Andrade and Laros (2007), Laros and Marciano (2008) and Bassetto (2019). In this same methodological approach, the results of studies by Riani and Rios-Neto (2008) and Brooke et al. (2014) indicated that male students of black descent perform less well than others. In addition, using the same methodological resource, Barbosa and Fernandes (2001) found that the father's schooling positively influences the student's proficiency, while the effect of the school gap is negative.

In another study along the same lines, Ferrão et al. (2001) used two-level hierarchical models (student and school) to analyze proficiency data in Mathematics, Portuguese, Science, Geography and History for students in the 4th grade of elementary school obtained from the 1999 SAEB. Their results highlighted that the student's socio-economic status is a factor that strongly influences performance in the subjects mentioned, as well as demonstrating that students who are behind in school tend to have lower academic results than those who are of the appropriate age for the grade. In addition, they identified that students of black descent showed lower performance in all geographical regions. The authors also found that schools with superior infrastructure, safety and cleanliness have a positive impact on student performance, while full-day students showed no advantage over part-time students.

Using data obtained from the Minas Gerais Public Education Assessment System (SIMAVE), Soares (2005) and Soares and Andrade (2006) used multilevel modeling to investigate the factors associated with students' school performance. A common result in these studies refers to the positive effect of socioeconomic status on student proficiency.

Soares (2005) showed that each year of school delay produces a drop of six points in proficiency, and that the fact that the student is male and black produces drops of nine and 10 points, respectively, in proficiency. While the frequency of teacher absences and lack of student motivation produce a drop in proficiency, the greater the teacher's dedication and availability, the higher the student's proficiency. In classes where students do their homework, there are also higher yields.

The estimates made by Brooke et al. (2014) corroborate the conclusions of Ferrão et al. (2001) and Soares (2005) when they state that prior proficiency and socio-economic level lead to better performance in Mathematics and Portuguese Language, male students are at a disadvantage compared to females when it comes to learning to read, black students and disadvantaged students have lower proficiency in both subjects.

Jesus and Laros (2004) used factor analysis to reduce the number of variables and found that socioeconomic status and the father's schooling have a strong influence on student performance in Portuguese. The authors also concluded that doing homework contributes positively to performance in Portuguese, while falling behind in school and the fact that the student works has a negative influence. Similarly, in addition to the mother's schooling, doing homework was a significant variable in explaining student performance in the studies by Machado et al. (2008), Riani and Rios-Neto (2008) and Moreira and Jacinto (2013).

The results found by Andrade and Laros (2007) show that students living in poorer housing, health and food conditions have characteristics that make learning more difficult.

Comparing the student with their peers, the cultural resources available to the student, whether the student enjoys studying the subject and does their homework are all variables that have a positive effect on school performance. On the other hand, lateness to school was identified as the variable that most negatively affected the student's school performance, followed by whether the student works, a result similar to those found by Barbosa and Fernandes (2001) and Soares (2005).

Laros, Marciano and Andrade (2010, 2012) also studied the factors that affect student performance. They proposed a multilevel model with data from the 2001 SAEB and considered the empirical model presented by Jesus and Laros (2004), with two levels, i.e. student and school. According to the results, the variable that most affected students' school performance was school delay, followed by whether the student likes studying math. The number of absences and the repetition rate negatively affected academic performance.

Unlike the other studies cited, Laros, Marciano and Andrade (2010) suggest interventions that the family and school can take to increase student performance. As far as the family is concerned, the authors highlight encouraging study from an early age through games and playful activities and preventing children from entering and staying at school with an age-grade distortion. Studies carried out in Brazil on educational determinants emphasize the role of the family in children's education, showing that family factors, especially parents' level of education, influence school performance.

Organizing and exploring the data

According to Menezes (2001), the School Boards are defined as direct administration units subordinate to the São Paulo State Department of Education and were created with the justification of eliminating the waste of human resources and promoting the efficient application of financial resources. In the state of São Paulo there are a total of 91 School Boards distributed throughout the state, covering areas of Greater São Paulo, the capital and other parts of the state. Spread across the state of São Paulo, there are a total of 63 school boards. In this study, data was taken on students enrolled in state public schools belonging to the DEs that make up the northern region of the state, i.e. Araraquara, Barretos, Franca, Jaboticabal, Ribeirão Preto, São Carlos, São Joaquim da Barra, Sertãozinho and Taquaritinga.

In the 2013 edition, 253,845 students enrolled in schools in the state of São Paulo. Of this total, students who did not answer the questionnaire applied by the assessment were excluded from the database structured for this research, as were those whose parents also did not answer the socio-economic questionnaire. Similarly, schools where coordinators and principals did not answer the questions relevant to this study were excluded. With these exclusions, the final database was made up of information on 106,120 students, 9,095 of whom attend schools whose DEs are linked to the Northern region. For these students, Table 2 presents descriptive information on the proficiencies achieved in Mathematics.

Table 2.

Performance in Mathematics in the northen region of the state of São Paulo in 2013

Level	Minimum	Median	Average	Maximum	Standard deviation	Percentage of students
AB	147.1	251.3	246.6	274.9	22.2	44.2%
BA	275.0	311.8	311.7	349.9	19.5	46.4%
AD	350.0	364.7	367.4	399.9	12.9	8.8%
AV	400.0	408.5	409.5	427.0	6.9	0.6%

It can be seen that in the Below Basic (AB) category, the lowest score was 147.1 points and the highest was 274.9 points. There are 44.2% of the students considered in this study at this level of proficiency, more than half of the students are in the lowest category. The standard deviation is 22.2 points, indicating that, when compared to the overall average - 246.6 points - students may have scored 22.2 points more or less. This data may indicate a high level of inequality in the scores of the same group of students. The range (difference between the highest and lowest score) for this group confirms the imbalance, at 127.8 points.

In the Basic (BA) category, 46.4% of the students were in the sample surveyed. Among the students classified in this space, the students with the lowest and highest scores, respectively, reached 275.0 and 349.9 points. The average for this group is 311.7 points. The standard deviation was 20.5 points, which indicates that students could score more or less than 19.5 points above the group average. The amplitude of the set was 74.9 points, indicating considerable inequality, but already lower than the AB group.

The group classified as Adequate (AD) includes students who achieved grades in line with what is expected in the year in which they are enrolled. In this category, the group analyzed had a minimum and maximum score of 350.0 and 399.9 respectively. The number of students in the sample who are in this group is 8.8%. The standard deviation of 12.9 points in this category is lower than in the previous ones, and is considered positive, as it indicates a variation in grades between 12.9 points more or less than the average of 364.7. The difference between the highest and lowest scores (range) was 49.9 points.

The last category deals with students at the advanced level (AV), who have mastered more than the content, skills and competencies of the year/grade they are in. They represent only 0.6% of the students in the sample analyzed. Within this level, the minimum and maximum scores are 400.0 and 427.0 points respectively. The average is 409.5 points. And the standard deviation is the lowest of all the categories, at 6.9 points, which represents little variation between the grades compared to the group average.

From the values shown in Table 2, it can be concluded that, within the group surveyed, approximately 90% of the students were classified in the AB and BA categories, showing the high percentage of students with insufficient mastery of the school content expected in the year/grade they are in.

Questionnaires and selection of variables

Considering the scope of the questionnaires applied by SARESP, which include a significant number of variables in the questionnaires for students, parents, principals and coordinators, it was necessary to select the most relevant variables. To identify which variables would be relevant, we used studies with similar objectives to this research. The variables that were most relevant to math performance were then identified.

The Table 3 identifies the variables and the associated authors.

Table 3.

Authors and variables considered in the hierarchical models.

Authors	Variables
Fletcher (1998)	Characteristics of the school and family environment, such as socioeconomic status, ethnicity and gender.
Barbosa and Fernandes (2000)	Father's schooling, gender, type of network.
Barbosa and Fernandes (2001)	Gap in schooling, father's schooling, motivation (likes math), home-school relationship, teacher's didactics and schooling, school infrastructure and equipment (conservation of the building, laboratories, furniture and equipment, school network).
Ferrão et al. (2001)	Socio-economic status, school delay, ethnicity, infrastructure, school safety and cleanliness, class shift.
Natis (2001)	Gender, school delay, grade (1st to 4th and 1st to 8th), performance of the principal.
Soares and Andrade (2006)	Socio-economic status, mother's schooling, school failure, school dropout, ethnicity, attendance at religious services.
Soares (2005)	Socio-economic status, gender, years of school failure, approximate number of teacher absences during the year, whether or not the teacher gives homework, class shift.
Jesus and Laros (2004)	Socio-economic status, father's schooling, ethnicity, school delay, homework, teaching resources.
Andrade and Laros (2007)	Socio-economic status, ethnicity, gender, homework, school delay.
Machado et al. (2008)	Failing grades, mother's schooling, homework, having more than 20 books, cost of pupils at school, class size, teacher training, presence of laboratories, school infrastructure.
Riani and Rios-Neto (2008)	School discrepancy, gender, ethnicity, rural and urban areas, mother's schooling, boss's occupation.
Brooke et al. (2014)	Gender, homework, school gap, parents' level of education, family income.

Based on the studies shown in Table 3, the questions were selected from the questionnaires applied to students, students' parents, coordinators and principals. Given the qualitative nature of all these questions, it was necessary to categorize and code them in order to carry out the modeling, as shown in Table 4. Questions Q1 to Q7 are answered by students, Q7 to Q12 by students' parents, Q13 to Q17 by coordinating teachers and Q18, Q19 and Q20 by school principals.

Table 4.

Select questions, categorization and coding of variables.

Question	Categorization*	Coding
Q1: What is the father's color or race?	0 - other (brown/mulatto, yellow, indigenous); 1 - white; 2 - black.	corp
Q2: What is the mother's color or race?	Same as Q1.	corm
Q3: Check the father's work situation.	0 - does not work (retired, unemployed, other situation); 1 - works (employed, self-employed, other).	trabp
Q4: What is the father's level of schooling?	0 - did not study; 1 - incomplete or complete schooling; 2 - incomplete or complete schooling; 3 - incomplete or complete schooling.	escp
Q5: Check the mother's work situation.	Same as Q3.	trabm
Q6: What is the mother's level of schooling?	Same as Q4.	escm
Q7: What is your household income in BRL?	0 - up to BRL 2,126.00; 1 - more than BRL 2,126.00.	rendaf
Q8: How many years have you been attending secondary school?	0 - no gap (3 years or less); 1 - gap (4 years or more).	def_EM
Q9: After the assessments, did you attend remedial classes to improve your results?	0 - yes (always, sometimes); 1 - no (I didn't need to, the school didn't offer it).	rec_mat
Q10: How much time do you spend doing your homework each day?	0 - you don't do any homework; 1 - you do homework (less than 20 mins, between 20 and 40 mins, between 20 and 40 mins), between 20 and 40 minutes, between 40 minutes and 1 hour, more than 1 hour).	lic_casa
Q11: Did you work outside during secondary school?	0 - no work; 1 - work (all the time, less than 1 year, 1 to 2 years, 2 to 3 years).	trab
Q12: Do you consider yourself:	Same as Q1.	cora
Q13: How many years have you been in this position/function?	0 - 10 years or less; 1 - more than 10 years.	tcargoc
Q14: Apart from this job, do you do any other work?	1 - yes; 0 - no.	outra_ativc
Q15: Are there any projects to reduce dropout rates at your school?	1 - yes; 0 - no.	tx_aband
Q16: Are there any projects to reduce failure rates at your school?	Same as Q15.	tx_reprov
Q17: Have you had a teacher shortage this year?	1 - yes; 0 - no.	faltaprof
Q18: How many years have you held this position?	Same as Q13.	t_cargod

Q19: Apart from this position, do you have any other?	1 - yes; 0 - no.	outrativd
Q20: What is your main activity?	0 - bureaucratic or administrative related to school management; 1 - pedagogical or serving the school community	princ_ativ

** Answers "I don't know" or "I don't want to answer" were disregarded.*

After categorizing the selected variables and in order to assess their significant influence on mathematics performance, multiple linear regression analyses were carried out. At the same time, the Stepwise[5] method was adopted as the criterion for selecting the model best suited to explaining the data. At this stage of the research, it is not intended to carry out an exhaustive interpretation of the parameter estimates associated with each variable, since the main focus is to assess the relevance of each variable in the model. The modeling and all the estimates presented in this study were conducted using the RStudio[6].

Considering all the variables related to the issues described in Table 4, a complete model was initially drawn up and is shown in expression (1).

$$\begin{aligned} profic_{ij} = \alpha + \beta_{1i} corp_i + \beta_{2i} corm_i + \beta_{3i} trabp_i + \beta_{4i} escp_i + \beta_{5i} trabm_i + \beta_{6i} escm_i + \\ \beta_{7i} rendaf_i + \beta_{8i} def\_EM_i + \beta_{9i} rec\_mat_i + \beta_{10i} lic\_casa_i + \beta_{11i} trab_i + \\ \beta_{12i} cora_i + \beta_{13i} t\_cargoc_j + \beta_{14i} outra\_ativc_j + \beta_{15i} tx\_aband_j + \\ \beta_{16i} tx\_reprov_j + \beta_{17i} falta\_prof_j + \beta_{18i} t\_cargod_j + \beta_{19i} outra\_ativd_j + \\ \beta_{20i} princ\_ativ_j + e_{ij} \end{aligned} \quad (1)$$

In this model, i and j denote the student and the school respectively. The parameter α characterizes the intercept, i.e. the expected value of proficiency in Mathematics when all the independent variables are equal to zero. $\beta_1, \beta_2, \ldots, \beta_{20}$ are the parameters associated with the variables and, when significant, indicate the effect of each one on the student's proficiency. The terme_{ij} is the random error and represents the variation in proficiency that is not explained by the variables included in the model.

The Table 5 shows the estimates for the parameters of the model selected by the Stepwise method, starting from the one given in expression (1)[7].

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[5] It is a variable selection technique used in statistical models to identify the independent variables that best explain the dependent variable (Draper and Smith, 1998).

[6] RStudio is an open source integrated development platform for the R programming language, widely used in statistical analysis and graphics generation. More information can be found at https://posit.co/products/open-source/rstudio/.

[7] In the tables containing the estimates, the symbols *, ** and *** represent the 10%, 5% and 1% significance levels, respectively.

Table 5.

Significant variables identified by Stewise method.

Variables	Estimates	Standard error	Variables	Estimates	Standard error
Intercept	258.475 ***	5.258	escm1	4.895	3.257
corp0	-2.598 *	1.160	escm2	10.081 **	3.353
corp2	-4.453 **	1.620	escm3	12.747 ***	3.701
corm0	-4.191 ***	1.148	rendaf1	5.430 ***	1.129
corm2	-8.448 ***	1.908	def_EM1	-3.175 *	1.577
trabm0	2.665 *	1.061	lic_casa1	9.903 ***	1.499
escp1	6.357 *	3.049	falta_prof0	-5.318 **	1.824
escp2	11.487 ***	3.167	princ_ativ0	-5.318 ***	1.824
escp3	15.178 ***	3.622			

According to the estimates shown in Table 5, the significant variables for explaining the math proficiency of public school students enrolled in the 3rd grade of secondary school in the northern region of the state of São Paulo: father's color (corp0 and corp2), mother's color (corm0 and corm2), whether the mother works or not (trabm0), father's schooling (escp1, escp2 and escp3), mother's schooling (escm2 and escm3), family income (rendaf1), GPA (def_EM1), whether or not the student does math homework (lic_casa1), whether there is a lack of teachers for any subject (falta_prof0) and the coordinator's predominant activity (princ_ativ0).

Given the significance of the variables in explaining student proficiency, they were all considered in the multilevel modeling proposed in this research.

Application of hierarchical models to SARESP data

In educational research, the population of interest comprises both the schools and the students attending these institutions. Sampling takes place in two distinct stages: initially, a sample of schools is selected, followed by the selection of a sample of students within each school. In the context of this study, the schools were grouped on the basis of the geographical regions of the state of São Paulo, as defined by the School Boards to which these schools are linked.

The hierarchical structure of the data is composed of two levels, in which students (considered the micro unit or level 1) are grouped into schools (considered the macro unit or level 2). The main objective of this research is to examine the impact of school-related variables on students' math proficiency. Following the multilevel modeling steps proposed by Raudenbush and Bryk (2002), we first estimated the null model, so called because it has no explanatory variables, and consists only of the intercept and an error term. This model plays a fundamental role in studying the distribution of the total variance of the response variable across the levels of the hierarchy. It is through the null model that the effect of the school on the student's math proficiency is estimated. In other words, the null model serves as the basis for estimating the explained variance versus the unexplained variance compared to the conditional models estimated a posteriori.

Assuming that the subscript *i* corresponds to the student and *j* the subscript represents the school, and considering the existence of J schools, $j = 1,...,J$, each with n_j students, $i = 1,...,n_j$, the null model can be specified as in expression (2).

$$\begin{aligned} y_{ij} &= \beta_{0j} + e_{ij} \\ \beta_{0j} &= \gamma_{00} + u_{0j} \end{aligned} \qquad (2)$$

where y_{ij} is the response variable (math proficiency) of the *i*-th student in the *j*-th school. It is assumed that $e_{ij} \sim N\left(0, \sigma^2\right)$ and $u_{0j} \sim N\left(0, \tau_{00}\right)$, and independence between the fixed effect and the random effects, as well as between the random effects.

According to the null model, the proficiency of student *i* studying at school *j* is defined by the intercept β_{0j} and the error term e_{ij}. The intercept β_{0j}, which represents a specific effect of school *j* on the student's math proficiency, is given by the sum of γ_{00} and u_{0j}, which indicate, respectively, the overall average math proficiency of students in the 3rd grade of secondary school - therefore, a fixed effect - and the distance of the average math proficiency of school *j* from the overall average γ_{00}.

Also in model (2), the first line characterizes level 1, which is associated with a random term represented by e_{ij} whose variance indicates the variability within schools, the second line is level 2, for which u_{0j} represents the random effect e, whose variance indicates the variability between schools. So, the variance in math proficiency is broken down into the variance between schools $\left(\tau_{00}\right)$ and within (or intra) schools $\left(\sigma^2\right)$ and, for this reason, the model with only the random intercept is called the variance components model. When inserting a variable at level 1 of the model, the associated coefficient can be treated as fixed or random. In the random case, $\mathbf{u}_j = \begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim NMV\left(\mathbf{0}, \mathbf{\Omega}_\mathbf{u}\right)$, with $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and variance-covariance matrix given by $\mathbf{\Omega}_\mathbf{u} = \begin{pmatrix} \sigma^2_{u_0} & \\ \sigma^2_{u_{01}} & \sigma^2_{u_{01}} \end{pmatrix}$.

The intra-school correlation coefficient (ICC) can be calculated from the estimates of variance within $\left(\sigma^2\right)$ and between schools $\left(\tau_{00}\right)$. This measures the proportion of variance between schools in relation to the total variance of the response variable, making it possible to measure the magnitude of the school effect, i.e. the heterogeneity of school results between schools. The ICC ranges from zero to one and is calculated using the ratio (3) (Kreft and De Leeuw, 1998; Raudenbush and Bryk, 2002; Goldstein, 1995, Gelman, 2006).

$$\rho = \frac{\tau_{00}}{\tau_{00} + \sigma^2}, \; 0 \leq \rho \leq 1. \tag{3}$$

When $\rho = 0$, the schools are homogeneous among themselves and, consequently, the student's performance is independent of the school in which they are enrolled. This means that all the variability in math proficiency is due only to differences between students, but not between schools. On the other hand, when $\rho = 1$, the variability in math proficiency depends only on the school in which the student is enrolled, i.e. the individual characteristics of the students have no effect on their school performance.

Bliese (2016) argues that, in addition to calculating the ICC, it is important to determine the significance of the variance τ_{00} and, to this end, suggests comparing the result of the Deviance, calculated by $-2log\ likelihood$, for two models, one containing the random intercept and the other not (Raudenbush and Bryk, 2002). The magnitude of the differences in Deviance between the models should be compared. A significant difference between these results suggests the existence of a relevant variation in the intercept in terms of mathematics proficiency between schools and, therefore, there is evidence to consider the model containing the random intercept. This criterion suggests that the model with the lowest Deviance value is the most appropriate (Kreft and De Leeuw, 1998). After calculating the ICC and verifying the significance of this value, variables must be introduced into the model.

For the northern region of the state of São Paulo, the ICC value was $\rho = 0{,}081$, suggesting that the percentage of variability contained in the student's math proficiency corresponds to approximately 8%.

The Table 5 shows that the significant variables associated with level 1 (student) to explain the performance in Mathematics achieved in the 2013 SARESP were the mother's and father's color and schooling, the mother's work situation, the gap in high school, family income and whether she does her homework. Considering these variables, different hierarchical models were proposed and, using the Akaike Information Criterion (AIC) (Bozdogan, 1987), the most appropriate one was selected to explain the performance of students from the northern region of the state in the 2013 SARESP.

The model selected was the one that considered, in addition to the intercept, the slope coefficient associated with the father's ethnicity as random, suggesting that, given a significant variance for this variable, the fact that the father is white, black or of another ethnicity could impact the child's performance in different ways between schools. Level 2 of the hierarchical model included variables indicating whether there is a shortage of teachers for any subject and the principal's main activity.

Among the fixed-effect and random-effect models proposed, the one corresponding to the lowest AIC value was selected and is shown in expression (4).

$$
\begin{aligned}
&level\ 1:\\
&y_{ij} = \beta_0 + \beta_{1j} corp_i + \beta_{2j} corm_i + \beta_{3j} trabm_i + \beta_{4j} escp_i + \beta_{5j} escm_i + \beta_{6j} rendaf_i\\
&\qquad + \beta_{7j} \mathrm{def}\ \_EM_i + \beta_{8j} lic\_casa\\
&level\ 2:\\
&\beta_{0j} = \gamma_{00} + \gamma_{01j} falta_{prof\ j} + \gamma_{02j} princ_{ativ\ j} + u_{0j}\\
&\beta_{kj} = \gamma_{k0}, k = 1, \dots, 8\\
&\gamma_{0lj} = \tau_{l0}, l = 1,2
\end{aligned}
\tag{4}
$$

For the selected model, the estimates obtained are presented and discussed in the next section.

Results and discussion

For the selected model, given in expression (4), the estimates are shown in Table 6. Considering these estimates, it is also valid to state empirically that students who are the children of parents who declare themselves to be white, who have a higher level of education, whose family income is higher than BRL 2,126.00 and who do their homework, obtained better scores in the 2013 SARESP compared to students who do not belong to the categories mentioned.

The Table 6 also shows that, according to the estimate of the intercept, the average score of the students from all the schools not belonging to the group with the best results was close to 259 points.

Table 6.

Estimates for the selected model.

Fixed effect					
Parameters	**Estimates**	**P-value**	**Parameters**	**Estimates**	**P-value**
Intercept	258.7416 5.5997	0.0000	escm1	5.0597 3.1840	0.1121
corp0	-2.1827 1.1422	0.0561	escm2	10.9309 3.2827	0.0009
corp2	-3.6642 1.5898	0.0212	escm3	11.9512 3.6243	0.0010
corm0	-3.2953 1.1286	0.0035	rendaf1	4.9604 1.1094	0.0000
corm2	-7.3638 1.8717	0.0001	def_EM1	12.5941 1.5507	0.0944
trabm0	3.2907 1.0404	0.0016	lic_casa1	9.0342 1.4859	0.0000
escp1	7.3236 2.9850	0.0142	falta_prof1	-4.6668 3.3465	0.1643
escp2	12.5261 3.1052	0.0001	princ_ativ0	-5.3021 2.6695	0.0480
escp3	14.8877 3.5519	0.0000			
Random effects					
	Variance	**Standard deviation**			
Intercepto					
Residual	2,038.27	45.15			

The father's and mother's level of education played an important role in the average math score achieved by their child in the 2013 SARESP, as all the estimates associated with these variables were significant. Table 5 shows that higher estimates were obtained for fathers and mothers with higher education, suggesting that levels of education provided better results in the assessment. Parents with higher education, complete or incomplete, provided an increase of almost 15 points in their child's Math average. Students with no school delay and who do their homework belong to the group of those with higher grades in Mathematics in the 2013 edition of SARESP. For those with no school delay, they had increases of almost 13 points.

A significant level 2 variable for student performance in math, in all the years analyzed, was the one that indicates the type of activity of the school principal, i.e. whether the activities are bureaucratic or administrative, or whether they are pedagogical or aimed at serving the school community. The estimates suggest that schools in which the principal has pedagogical activities contributed negatively to the results of the 2013 SARESP, as the estimates were negative.

According to the model selected, the effect of the variable indicating the mother's level of schooling on the student's performance is given by $1,9743 + \widehat{u_{5j}}$ for mothers who have studied up to elementary school, $7,4090 + \widehat{u_{5j}}$ for mothers who have secondary schooling, and $12,2864 + \widehat{u_{5j}}$ for those who have completed or incomplete higher education. For each level of education of the mother, a coverage interval can be calculated for the estimates of the slope coefficient associated with that variable. Considering the estimated variances for escm1, escm2 and escm3, the intervals corresponding to mothers with primary education, secondary education and higher education are given, respectively, by $1{,}9743 \pm 1{,}96\sqrt{245{,}13}$, $7{,}4090 \pm 1{,}96\sqrt{259{,}40}$ and $12{,}2864 \pm 1{,}96\sqrt{347{,}88}$, resulting in the intervals (28,71; 32,66), (-24,16; 38,98) and (-23,70, 48,84).

Therefore, it can be empirically stated that, for the average school, an increase in the student's grade of almost 2 points is expected if the mother's level of schooling is elementary school, 7.4 points for students whose mothers have secondary schooling, and 12.3 points if the mother has higher education. However, with the coverage intervals for the slopes of the mother's level of schooling, it is estimated that in 95% of the schools, the increase in the grade of a student whose mother has primary schooling is approximately between 28.7 and 32.7 points, for those whose mother has secondary schooling the increase will be a maximum of 39 points, and for students whose mothers have higher education, the maximum increase in the grade will be approximately 49 points, leading to different averages in mathematics between the schools. Although negative values are found in the coverage intervals for mothers with secondary and higher education, they are meaningless when interpreting the increase in points in the student's average. All comparisons were based on the scores of students with mothers with no schooling, for whom the average math score in SARESP was close to 262.8 points.

Final Considerations

Although it is in the process of being developed in Brazil, large-scale educational assessment has proved to be an important tool in recent decades, helping to identify factors that affect student performance at school. In recent years, the priority objective of assessment systems has been to find mechanisms that can effectively and efficiently improve the quality of education offered in society. In turn, educational evaluation aims to provide elements that make it possible to diagnose the situation of the educational system in a given location and, at the same time, to subsidize appropriate policies and guidelines in the municipal, state and national context, with a view to continuously improving the quality of education.

It is well known that students' school performance, measured by their proficiency in standardized tests, is the result of a complex interaction of factors that act concomitantly at the various levels of their social insertion, i.e. socio-economic aspects in the family environment, relationships and teaching practices in the school environment, among others. With this in mind, the aim of this study was to identify the factors, both internal and external to the school, that can influence students' educational performance. To this end, SARESP data from 2013 was used. The information refers to the grades obtained by the students in the Mathematics test and the questionnaires applied by SARESP to the students and parents of the students who took part in the assessment, and those applied to the schools, in which the coordinating teachers and principals answered the questions posed. The database used in this research was organized and composed according to the school districts to which the schools belonged, prioritizing those located in the northern region of the state of São Paulo.

Next, a linear regression was carried out involving all the previously selected variables in order to identify those that were significant in explaining student performance. Estimates for the parameters of the regression model were obtained using RStudio software and, using the Stepwise procedure, the model best suited to the data and which best explained the phenomenon of interest was selected.

With the significant variables to explain math performance, we began to propose two-level hierarchical models, with level 1 associated with the student and level 2 with the school. The motivation for using multilevel modeling is due to the fact that the data obtained is hierarchically structured, students are placed in schools, making it possible to affirm that school performance is determined by various factors (Barbosa and Fernandes, 2001; Soares, 2005; Palermo, Silva and Novellino, 2014).

Among the advantages of multilevel models is the fact that, for hierarchically structured data, these models make it possible to investigate the influence of the characteristics of each level of the hierarchy on students' school performance and the differentiation between schools. At the same time, these models make it possible to separate the variability in results associated with schools from the variability within each school, observed among students.

The results show that the factors that have the greatest impact on school performance come from the student's background, i.e. family aspects such as income, ethnicity and the parents' work situation, whether the student is behind in school, whether they do their homework, among others. However, it is possible to say that the school also has characteristics that directly or indirectly influence the student's school performance.

Studies like this seek to gain knowledge to support solutions to problems that are a constant concern for all those involved in the education system. Therefore, we hope to have contributed to the literature that addresses the identification of factors that impact student educational performance and highlight the importance of formulating and implementing public education policies that simultaneously improve the quality of education offered to society and reduce the impact of socioeconomic characteristics on student performance.

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