EMP Educação Matemática Pesquisa
ISSN 1983-3156
Qualis A1
http://dx.doi.org/10.23925/1983-3156.2025v27i3p054-073

Explorando as propriedades da igualdade: uma tarefa de aprendizagem profissional na formação inicial de professores de matemática

Exploring the properties of equality: a professional learning task in the pre-service of mathematics teachers' education

Explorando las propiedades de la igualdad: una tarea de aprendizaje profesional en la formación inicial de profesores de matemáticas

Explorer les propriétés de l'égalité : une tâche d'apprentissage professionnel dans la formation initiale des enseignants de mathématiques

Samuel Ribeiro da Silva[1]
Universidade Federal Rural de Pernambuco
Mestre em Ensino de Ciências e Matemática
https://orcid.org/0000-0003-1363-5251

Jadilson Ramos de Almeida[2]
Universidade Federal Rural de Pernambuco
Doutor em Ensino de Ciências e Matemática
https://orcid.org/0000-0003-3707-4807

Abstract

This article investigates the mathematical and pedagogical knowledge of future mathematics teachers regarding the properties of equality, using the Mathematics Teacher's Specialized Knowledge (MTSK) framework. We adopted a formative process guided by a Professional Learning Task (PLT), which also served as the basis for qualitative data collection. The analysis followed the method of systematic comparisons. The study involved six participants who were enrolled in the Supervised Teaching Practicum III course at a public university in the state of Pernambuco, Brazil. The results indicated that the pre-service teachers demonstrated early signs of mastering key MTSK subdomains, particularly KoT (knowledge of properties, procedures, and representation registers) and KPM (knowledge of proper use of symbols and mathematical justifications), both of which are essential for teaching equality properties. Their active engagement in discussions and reflection during the formative sessions fostered the expansion of their professional knowledge. These findings highlight the potential of Professional Learning Tasks (PLTs) to create meaningful opportunities for professional learning and to support the professional development of future mathematics teachers during their initial training.

[1] samuel.ribeiro.@ufrpe.br
[2] jadilson.almeida@ufrpe.br

***Keywords**:* Initial teacher education, MTSK, Properties of equality, Professional learning task.

Resumen

Este artículo identifica los conocimientos matemáticos y didácticos que poseen los futuros profesores de matemáticas acerca de las propiedades de la igualdad, a la luz del marco Mathematics Teacher's Specialized Knowledge (MTSK). Para eso, se implementó un proceso formativo mediado por una Tarea de Aprendizaje Profesional (TAP), que asimismo funcionó como instrumento para la producción de datos cualitativos. El análisis se realizó mediante el método de comparaciones sistemáticas. Participaron seis estudiantes de la asignatura Práctica Docente Supervisada Obligatoria III de una universidad pública de Pernambuco, Brasil. Los resultados muestran indicios tempranos de dominio en los subdominios KoT (conocimiento de propiedades, procedimientos y registros de representación) y KPM (conocimiento del uso adecuado de símbolos y justificaciones matemáticas), ambos fundamentales para la enseñanza de las propiedades de la igualdad. La participación de los licenciandos en las discusiones y reflexiones del proceso formativo amplió sus conocimientos profesionales, lo que evidencia el potencial de las TAP para generar oportunidades de aprendizaje y fomentar el desarrollo profesional en la formación inicial del profesorado de matemáticas.

***Palabras clave**:* Formación inicial, MTSK, Propiedades de la igualdad, Tarea de aprendizaje profesional.

Résumé

Cet article identifie les connaissances mathématiques et didactiques que possèdent de futurs enseignants de mathématiques à propos des propriétés de l'égalité, dans le cadre théorique du Mathematics Teacher's Specialized Knowledge (MTSK). Un dispositif de formation, médié par une Tâche d'Apprentissage Professionnel (TAP), a simultanément servi à recueillir des données qualitatives. L'analyse a été menée au moyen de la méthode des comparaisons systématiques. Six étudiants inscrits à l'unité d'enseignement Stage supervisé obligatoire III d'une université publique du Pernambuco, Brésil ont participé à l'étude. Les résultats mettent en évidence des manifestations précoces des sous-domaines KoT (connaissance des propriétés, des procédures et des registres de représentation) et KPM (connaissance de l'usage adéquat des symboles et des justifications mathématiques), essentiels à l'enseignement des propriétés de l'égalité. La participation active des licenciands aux discussions et aux réflexions durant la formation a enrichi leurs connaissances professionnelles, confirmant le potentiel des TAP pour générer des occasions d'apprentissage et favoriser le développement professionnel dans la formation initiale des enseignants de mathématiques.

Mots-clés *:* formation initiale, MTSK, propriétés de l'égalité, tâche d'apprentissage professionnel.

Resumo

Neste artigo, buscou-se identificar os conhecimentos matemáticos e didáticos de futuros professores de matemática, sobre as propriedades da igualdade, na perspectiva do *Mathematics Teacher's Specialized Knowledge* (MTSK). Para isso, acompanhamos um processo formativo mediado por uma Tarefa de Aprendizagem Profissional (TAP), a qual também serviu para produção dos dados qualitativos, sendo utilizado o método de comparações sistemáticas como procedimento de análise. Os sujeitos, em número de 6, estavam cursando a disciplina Estágio Supervisionado Obrigatório III em uma universidade pública do Estado de Pernambuco. Os resultados mostraram que os licenciandos já revelaram indícios dos subdomínios KoT (conhecimento das propriedades, procedimentos, registros de representação) e do KPM (conhecimento do uso correto dos símbolos e justificações matemáticas), ligados ao tema e ao ensino das propriedades da igualdade. A participação dos licenciandos, por meio das discussões e reflexões no momento formativo, permitiu a ampliação dos seus conhecimentos profissionais, corroborando, assim, as potencialidades da Tarefa de Aprendizagem Profissional (TAP) em gerar oportunidades de aprendizagem profissional e em promover desenvolvimento profissional na formação inicial de professores de matemática.

Palavras-chave*:* Formação inicial, MTSK, Propriedades da igualdade, Tarefa de aprendizagem profissional.

Exploring the properties of equality: a professional learning task in the pre-service of mathematics teachers' education

Several studies show that teacher education rooted in professional knowledge has emerged as a prominent research field in mathematics education over the past few decades, both in Brazil and abroad (Ferreira, 2003; Pazuch & Ribeiro, 2017; Ribeiro, Almeida & Mellone, 2021).

Other studies indicate that one of the primary challenges in mathematics teaching and learning is the gap between the one that is learned in universities and the one that is taught in schools (Ribeiro, 2019), a dilemma compounded by the persistent disconnect between theory and classroom practice (D'Ambrósio, 2012).

Furthermore, authors such as Ponte (2012), Ferreira, Ribeiro & Ribeiro (2017), and Oliveira & Fiorentini (2018) stress that teachers' mathematical and pedagogical knowledge is pivotal in shaping classroom practices and, by extention, student learning.

Drawing on these authors, as well as on Blanco (2003), we contend that professional learning and professional knowledge constitute two key dimensions of teacher professional development, serving as the basis for initiatives to improve the quality of teacher education.

Accordingly, drawing on Ribeiro (2019) and Ribeiro and Ponte (2020), we argue that pre-service mathematics teachers must be offered targeted learning opportunities to address the disparities identified in the literature. Integrating Professional Learning Tasks (PLTs) into formative programs is one promising way to achieve this goal.

We then focus our study on the mathematical object 'properties of equality', given its prominent place in the Base Nacional Comum Curricular (BNCC, Brazil 2018) and the documented difficulties teachers and students encounter in mastering this content (Teles, 2002; Lessa, 2005; Ponte, Branco & Matos, 2009; Trivilin & Ribeiro, 2015).

In light of these considerations, we pose the following research question: What mathematical and pedagogical knowledge do pre-service mathematics teachers possess—and further develop—for teaching the properties of equality during a formative experience mediated by a Professional Learning Task (PLT)?

Accordingly, this article has two aims: first, to identify the mathematical and pedagogical knowledge related to the properties of equality that pre-service teachers demonstrate while engaging in a PLT-mediated formative process, and second, to examine the potential of the PLT to create meaningful professional-learning opportunities within initial teacher education.

To achieve these objectives, we will first synthesize the literature on the properties of equality, then outline the Mathematics Teacher's Specialized Knowledge (MTSK) framework, describe our methodology with particular attention to the concept of Professional Learning Tasks and, finally, analyse the data in light of the theoretical and analytical framework just mentioned.

The Properties of Equality

We observe that, in recent years, the Base Nacional Comum Curricular (BNCC) has designated Algebra as a thematic area (Brazil, 2018), calling for the development of algebraic thinking from the earliest years of elementary school (Almeida 2016). The associated curricular proposals are intended to reduce the persistent difficulties that both pupils and teachers experience with algebra, as evidenced by discouraging results on metrics such as the Basic Education Development Index (IDEB), the assessments of the Basic Education Assessment System (SAEB) and the Program for International Student Assessment (PISA).

Within the Algebra strand, the BNCC singles out the mathematical object properties of equality, stipulating that related skills be introduced in the 3rd grade and progressively deepened through the 7th grade, when students begin work with first-degree equations (Brazil 2018).

For decades researchers have documented the obstacles learners and teachers encounter in grasping the equal sign and its associated properties (Kieran 1981; Teles 2002; Lessa 2005; Trivilin and Ribeiro 2015; Barboza 2019).

In this context, the studies of Teles (2002), Ponte, Branco and Matos (2009) and Oliveira and Fernández (2012) catalogue the principal properties of mathematical equality, which we summarize in Table 1.

Table 1.

Properties of Mathematical Equality

Property	Description	Example
Reflexive	If one number is equal to another, then the second is equal to the first.	If a = b, then b = a, for any numbers a and b.
Symmetric	A number is always equal to itself.	a = a, for any number a.
Transitive	If one number is equal to a second, and the second is equal to a third, then the first is equal to the third.	If a = b and b = c, then a = c, for any numbers a, b, and c.

Additive Principle	If two numbers are equal, then adding or subtracting the same quantity on both sides of an equation preserves equality.	$a = b \rightarrow a + c = b + c$ $a = b \rightarrow a - c = b - c$
Multiplicative Principle	If two numbers are equal, then multiplying or dividing both sides of an equation by the same non-zero quantity preserves equality.	$a = b \rightarrow a \,.\, c = b \,.\, c$ $a = b \rightarrow a/c = b/c \ (c \neq 0)$

In light of the above, we observe that the BNCC only briefly mentions the additive and multiplicative principles of equality only briefly in its curricular proposal, which we regard as a gap. This is because equality is a polysemic concept, its various meanings give rise to several kinds of conceptual difficulty (Ponte, Branco, & Matos, 2009). This situation underscores the need for further research on this mathematical object in algebra teaching and learning.

In the following section, we introduce the Mathematics Teacher's Specialized Knowledge (MTSK) framework, which underpins our study and guides the analysis of the empirical data.

The Mathematics Teacher's Specialized Knowledge (MTSK) Model

The MTSK framework emerged from two key antecedents: Shulman's (1986, 1987) knowledge base for teaching and the Mathematical Knowledge for Teaching (MKT) framework developed by Ball, Thames and Phelps (2008).

Shulman (1987) underscored the centrality of Pedagogical Content Knowledge (PCK) to effective teaching and learning. Although his knowledge base addressed teaching across disciplines, it was later reorganized and tailored specifically to mathematics education by Deborah Ball and her colleagues.

Building on that work, the SIDM research group at the University of Huelva, Spain, identified operational limitations in the subdomain structure of the Mathematical Knowledge for Teaching (MKT) framework. They refined the model and argued that all of a mathematics teacher's knowledge is specialized (Carrillo et al., 2014; Mello, Junior, & Wielewski, 2017). This effort produced the Mathematics Teacher's Specialized Knowledge (MTSK) framework, which comprises two overarching domains: Mathematical Knowledge (MK) for teaching and Pedagogical Content Knowledge (PCK). Both domains are depicted in Figure 1.

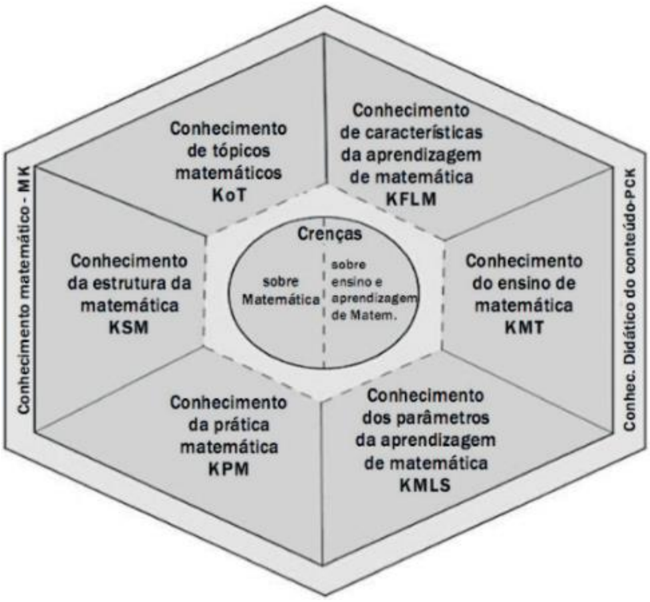


Figure 1.

MTSK (Carrillo et al., 2014, translated by Moriel Junior and Wielewski, 2017, p. 130)

These domains are further subdivided into three 'attention focuses'(Ribeiro, 2021). Within the MK domain, the subdomain KoT (Knowledge of Topics) comprises knowledge of mathematical content—including procedures, phenomenology, definitions, concepts, and representation registers. KSM (Knowledge of the Structure of Mathematics) encompasses understanding of interconceptual connections with prior and future mathematical content. KPM (Knowledge of Mathematical Practice) covers knowledge of proofs, justifications of procedures, and mathematical validation.

In the PCK domain, KFLM (Knowledge of Features of Learning Mathematics) addresses students' potential, difficulties, obstacles, and common errors related to specific content. KMT (Knowledge of Mathematics Teaching) concerns teaching strategies, including resources, tasks, scaffolding, and examples used in instruction. Lastly, KMLS (Knowledge of Mathematical Learning Standards) involves expected learning levels and standards for students, as defined by national and international official documents and other authoritative sources.

At the center of the model lies the 'Beliefs' domain, which encompasses beliefs about mathematics and about teaching and learning the subject; these beliefs permeate and influence all other subdomains.

Although the model presents clear subdivisions, the subdomains are inherently interconnected and complementary; the divisions serve solely analytical purposes (Carrillo & Martin, 2019; Ribeiro, 2022).

As noted earlier, this study employs the MTSK framework to categorize and analyze the data collected during the formative phase, as detailed in the next section.

Methodological Approach

This study adopts a qualitative design in the sense described by Bogdan and Biklen (1994): data are gathered in the natural setting, the researcher serves as the primary instrument, emphasis falls on process rather than product, and data are descriptive, inductively analyzed, and interpreted for meaning. We also follow Oliveira (2011), defining the object of study through its context, time frame, literature base, and data-collection procedures.

The formative process involved six pre-service mathematics teachers (hereafter PSMTs) from a public university in Pernambuco. All were enrolled in Supervised Teaching Practicum III, the program's penultimate-semester course. The course instructor (teacher educator in this study, represented as (TE)) helped monitor and guide the activities.

Although the class initially comprised 19 students, only six agreed to participate after the instructor's invitation.

Activities were carried out in two sessions jointly scheduled by the researcher and the instructor. Session 1 administered an exploratory questionnaire (not analyzed in this article). Session 2 implemented the Professional Learning Task (PLT) with the six participating students.

The PLT is an artifact designed to prompt both mathematical and pedagogical discussion, in line with the Professional Learning Opportunities for Teachers (PLOT) model proposed by Ribeiro and Ponte (2020).

It is important to highlight that the The researcher did not intervene in participants' discussions or in the plenary; rather, the researcher co-designed the PLT and overall formative structure with the instructor, distributed tasks during the sessions, and audio-recorded all discourse for later analysis.

According to Ribeiro and Ponte (2020), the PLOT model comprises three interconnected domains: PAF (Actions of the Teacher Educator), PLT (Professional Learning Tasks), and PDI (Potential Discursive Interactions), as shown in Figure 2.

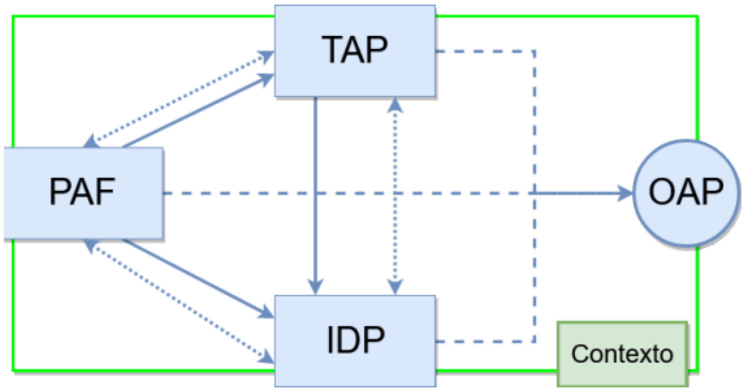


Figure 2.

PLOT Model, Ribeiro e Ponte (2020, p. 4)

According to these authors, the three domains of the PLOT model have both conceptual and operational dimensions, each with critical aspects.

a) In the PAF domain (Actions of the Teacher Educator), these aspects include bridging the gap between university and school mathematics, linking mathematics with pedagogy, creating an exploratory learning environment, and orchestrating mathematical and pedagogical discussions that foster teacher learning.

b) In the PLT domain (Professional Learning Tasks), they involve designing tasks that surface professional teaching knowledge, cultivating an exploratory classroom climate, selecting high-level cognitive challenges, and using practice artifacts, such as video recordings as learning resources.

c) In the PDI domain (Potential Discursive Interactions), they entail promoting mathematical and pedagogical dialogue, encouraging teachers to argue and justify during task discussions, employing precise disciplinary language, and nurturing dialogic communication between teachers and their students (Ribeiro & Ponte, 2020).

The PLOT model also unfolds in three operational phases. Phase 1, Organization, is the stage at which the educator designs the formative process, constructs the PLTs, and identifies potential PDIs. Phase 2, Development, is when the participants—the educator and the teacher-learners—interact as the PLTs are enacted and the PDIs emerge. Phase 3, Finalization, is the stage in which, through the convergence of PAF, PLT, and PDI, the professional learning opportunities (PLOs) are fully realized (Ribeiro & Ponte, 2020, p. 4).

In our study we followed these phases. During Phase 1 the course instructor and the researcher co-designed the PLT, which included a mathematical problem (MP) suitable for elementary students plus six items addressing mathematical and pedagogical knowledge aligned with the MTSK subdomains and focused on the properties of equality.

The problem was chosen on the basis of the grade levels in which the pre-service teachers were completing their practicum and BNCC guidelines for teaching the properties of equality, which spans from 3rd to 7th grade of elementary school.

In Phase 2 the MP was distributed to the six pre-service teachers who first solved it individually.

Because of space constraints, this article analyzes only their individual written solutions.

After those responses were collected, we began audio-recording the discursive interactions. Working in two groups (G1 and G2) of three students each, the pre-service teachers collaborated on the second part of the PLT. Phase 3 culminated in a plenary session in which the two groups, moderated by the instructor, discussed all PLT items together.

A PLT, according to Ribeiro and Ponte (2020), should be purposefully designed to help teacher-learners examine school mathematics tasks from both mathematical and pedagogical perspectives. Guided by that principle, we divided our PLT into two parts. Part 1 presented an elementary-level task (Figure 3) featuring a two-pan balance—a textbook image commonly used to illustrate the properties of equality (Teles, 2002). Such a metaphor highlights balance and equivalence, both essential for understanding equations (Lessa, 2006).

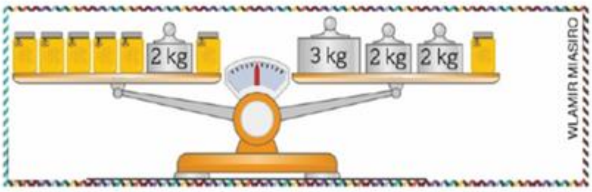
The pre-service teachers received the following prompt:

"Solve the problem by finding the weight of the pot, and justify how you arrived at your results."

Um professor de matemática apresentou a seguinte situação: "Na feira municipal, uma banca de doces, produzidos por agricultores da região, utiliza uma balança de dois pratos para indicar a quantidade de produto que o consumidor está comprando. A comerciante colocou potes de doces todos do mesmo formato e com a mesma quantidade em uma balança que está em equilíbrio, como mostrado na imagem."

O professor explicou à turma que a balança de dois pratos é um objeto que foi muito utilizado pelo comércio no passado, antes de surgirem as balanças digitais.

Então, o professor pediu para que os estudantes encontrarem o peso de cada pote de doce e, em seguida, perguntou o que aconteceria com a balança se: 1) a comerciante retirasse um peso de 3 kg do prato da direita; 2) a comerciante retirasse pesos de 2 kg de cada lado; 3) a comerciante retirasse um peso de 2 kg do prato da direita e um pote de doce do lado esquerdo.



Fonte: adaptado de Santos, Azevedo e Rodrigues (2020, p. 61)

Figure 3.

Mathematical task (Santos, Azevedo & Rodrigues, 2020, p. 61)

In the second part of the PLT, six items (Figure 4) were deliberately crafted to address both mathematical and pedagogical aspects, fostering discussion and reflection among the pre-service teachers.

<u>2ª parte: (em grupo)</u>

Quatro alunos resolveram a questão assim:

Um aluno respondeu que cada pote de doce tem 2 kg.

Outro falou que, se retirar um peso de 3 kg do prato da direita, a balança permanecerá em equilíbrio.

O terceiro afirmou que, se a comerciante tirar o peso de 2 kg do prato da direita e 2 kg do da esquerda, a balança volta a ficar em equilíbrio.

O quarto disse que, se a comerciante retirar um peso de 2 kg do prato da direita e um pote de doce da esquerda, a balança volta a ficar em equilíbrio.

a) Você concorda com as respostas dos quatro alunos acima? Justifique.

b) Em qual ano escolar essa questão poderia ser aplicada? Por quê?

c) Você faria alguma adaptação nessa atividade? Qual? Por quê?

d) Que recursos (físicos ou digitais) você usaria para ensinar essa tarefa aos alunos no ensino fundamental? Justifique.

e) Quais conceitos você identificou que a tarefa aborda?

f) Indique uma tarefa que você trabalharia com os alunos após executar a tarefa da balança.

Figure 4.

Second part of the PLT (authors)

With this information in hand, we now turn to the results of the written responses, audio transcripts, and data analysis.

Results and Data Analysis

As previously noted, and given space constraints, this article reports only the findings from the first part of the PLT, namely, the pre-service teachers' individual solutions to the mathematical task (MT) and the ensuing discussions in both the small-group work and the plenary session.

Table 2 summarizes the coding categories that emerged from these data: KoT (Knowledge of Topics) and KPM (Knowledge of Mathematical Practice).

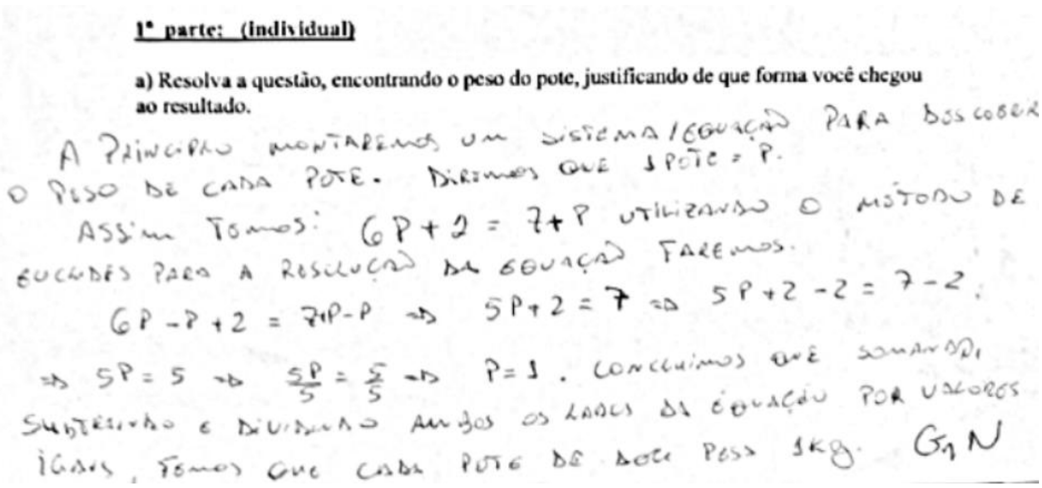
Table 2.

MTSK Categories Related to the Properties of Equality

MTSK Subdomains	Descriptors	Codes
KoT	Properties	KoT1
KoT	Procedures	KoT2
KoT	Representation registers	KoT3
KPM	Correct use of symbols	KPM1
KPM	Mathematical justifications	KPM2

We first observed that all pre-service teachers correctly determined the pot's weight and used letters to represent the unknown variable, providing evidence of KoT2 (procedural knowledge). However, not every participant justified the procedure or explained how the answer was obtained.

We also found that four pre-service teachers solved the mathematical task (MT) with the technique described by Almeida (2017): transposing terms from one side of the equation to the other by changing their signs (KoT2). Only two participants (PST2 and PST3) explicitly applied the properties of equality, demonstrating knowledge of both KoT1 (properties) and KoT2 (procedures). This is illustrated in the solution protocol of pre-service teacher 3 (Figure 5).



1ª parte: (individual)

a) Resolva a questão, encontrando o peso do pote, justificando de que forma você chegou ao resultado.

A PRINCÍPIO MONTAREMOS UM SISTEMA/EQUAÇÃO PARA DESCOBRIR O PESO DE CADA POTE. DIREMOS QUE 1 POTE = P. ASSIM TEMOS: $6P + 2 = 7 + P$ UTILIZANDO O MÉTODO DE EUCLIDES PARA A RESOLUÇÃO DA EQUAÇÃO FAREMOS.

$6P - P + 2 = 7 + P - P \Rightarrow 5P + 2 = 7 \Rightarrow 5P + 2 - 2 = 7 - 2$;

$\Rightarrow 5P = 5 \Rightarrow \frac{5P}{5} = \frac{5}{5} \Rightarrow P = 1$. CONCLUÍMOS QUE SOMANDO, SUBTRAINDO E DIVIDINDO AMBOS OS LADOS DA EQUAÇÃO POR VALORES IGUAIS, TEMOS QUE CADA POTE DE DOCE PESA 1KG. G1 N

Figure 5.

PST3's solution to the mathematical task (G1)

We found that PST3 explicitly invoked the properties of equality when solving the equation and explained each step of the method. This performance evidences KPM2 within the domain of mathematical practice, as characterized by Moriel Junior and Carrillo (2014), and aligns with Cabanha's (2018) distinction between merely knowing a procedure (KoT3) and being able to justify it (KPM2).

PST3 also used multiple representation registers—natural language, numerical language, and algebraic notation—thereby showing evidence of KoT3. In addition, PST3 demonstrated a clear grasp of the task's context and interpretation (KoT4) and employed mathematical symbols correctly, signaling maturity in mathematical practice (KPM) as described by Ribeiro, Almeida, and Mellone (2021).

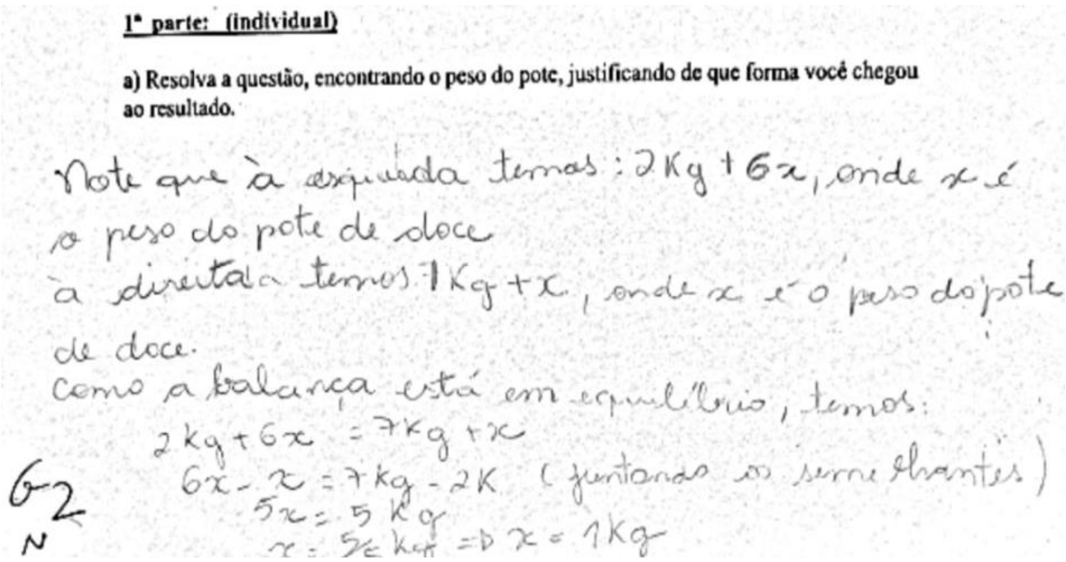
We conclude, therefore, that PST3 displays knowledge characteristic of algebraic thinking as defined by Almeida (2016): establishing relationships, operating with unknowns, and constructing meaning—elements that, in this case, fall under mathematical content knowledge (KoT).

In the group G1 discussion (excerpt G1 [9–11]), PST3 proposed an alternative solution they called “by observation.” We interpret this approach as reflecting the balance concept highlighted by Lessa (2005), a key factor in understanding the principle of equivalence when working with first-degree equations:

> 9 PST3: “I solved it algebraically. I let the pot’s weight be x, which gave the equation 6x+2=7+x6x + 2 = 7 + x6x+2=7+x; solving, x=1x = 1x=1. You can also see it by observation: remove 2 kg from each side, leaving five pots that together weigh 5 kg, so each pot weighs 1 kg. So there are two possible approaches—observation and algebra.”
> 10 PST1: *“I just used the algebraic method for this one.”*
> 11 PST2: “I set the pot’s weight as x, an unknown, and since the scale is balanced, the two sides are equal: 6x+2=7+x6x + 2 = 7 + x6x+2=7+x, because the right side has only one pot of candy. Solving, x=1x = 1x=1 kg.”

We therefore identified clear evidence that a Professional Learning Opportunity (PLO) occurred. When PST3 revealed that they had solved the task by a different method and explained their approach, the other pre-service teachers began to reflect on this alternative, potentially revising their own understanding or mathematical practices, as the plenary-session transcript suggests.

This episode exemplifies the kind of dialogic interaction that can trigger PLOs, consistent with Ribeiro and Ponte’s (2020) framework.


1ª parte: (individual)

a) Resolva a questão, encontrando o peso do pote, justificando de que forma você chegou ao resultado.

Note que à esquerda temos: 2Kg + 6x, onde x é o peso do pote de doce
à direita temos 7Kg + x, onde x é o peso do pote de doce.
Como a balança está em equilíbrio, temos:
$2kg + 6x = 7kg + x$
$6x - x = 7kg - 2K$ (juntando os semelhantes)
$5x = 5kg$
$x = \frac{5}{5}kg \Rightarrow x = 1kg$

G2
N

Figure 6.

PST4’s solution to the mathematical task – G2 (authors)

In Group G2 (Figure 6), PST4 likewise transposed terms to solve the equation but also pointed to the balance of the scale as the task's key insight, noting, "Since the scale is balanced …."

Although they did not explicitly name the properties of equality, PST4 recognized the balance of the scale and drew an analogy to the principle of equivalence (Lessa, 2005). This response provides evidence of KoT1 (knowledge of properties) and KoT2 (procedures), which Fiorentini, Fernandes, and Cristóvão (2005) regard as indicators of developing algebraic thinking.

PST4 also solved the task using three representation registers—natural language, numerical notation, and algebraic symbolism demonstrating KoT3, which Moriel Junior and Carrillo (2014) interpret as evidence of specialized teacher knowledge within mathematical content.

The audio recording of Group G2 shows that participants did not discuss their solution strategies; instead, they moved directly to item "a" in the second part of the PLT, which is beyond the scope of this article.

During the plenary session, the teacher educator (TE) asked the participants how they had approached the mathematical task, as shown in excerpt PL [13–32]:

13 PST1: "My colleague noticed that if you remove 2 kg and one pot from one side, and 2 kg and one pot from the other, taking the same weight off both sides, you're left with five pots equaling 5 kilograms. So each pot weighs 1 kg."

14 PST4: "We used a more algebraic approach. We let x represent the unknown weight of a pot, set up the equation 6x+2=7+x6x + 2 = 7 + x6x+2=7+x, combined like terms, and solved for x."

15 TE: "So you call it algebra because you introduced x. What exactly did x stand for?"

16 PST4: "The pot. Since there was just one pot on the right side, we called that pot x; its weight is 1x1x1x. On the left, there were six pots, also of weight x each. Because the scale is balanced, the two sides are equal, so we solved the equation for x"

17 TE: "Which operations did you perform to find the value of x?"

18 PST4: "First we subtracted x from both sides, then subtracted 2 kg from both sides, divided by 5, and found that each pot weighs 1 kg."

19 TE (turning to Group 1): "How did your group find the value?"

20 PST3: "I solved it algebraically at first, but I realized there's another way. If you recognize that 2 kg + one pot appears on both sides, you can remove them and keep the balance. Then you have five identical pots weighing 5 kg in total, so each pot is 1 kg. A student who doesn't yet know algebra could reach the answer by that kind of observation and manipulation."

21 TE: “Do you see anything similar tbetween your method and Group 2’s? Which ideas overlap?”
22 PST1: “Subtracting from both sides?”
23 TE: “Subtracting what from both sides?”
24 PST1: “The same quantity.”
25 TE: “In PST1’s case they’re using the original scale picture. Did you change representations?”
26 PST4: “In the picture, one side clearly shows 2 kg. I think PST1 noticed that the other side has 3 kg, 2 kg, and another 2 kg. So you removed one 2 kg weight, is that it?”
27 PST1: “Plus the pot.”
28 PST4: “Right! Plus the pot!”
29 PST1: “Yes, plus the pot.”
30 PST4: “So there were 5 kg left.”
31 PST1: “Exactly. After removing 2 kg and one pot from each side, we had five pots on one side, so we could assign a value to each.”
32 PST4: “Right, I get it now! You left only pots on one side. That’s really interesting!”

From this plenary-session excerpt, we observed that the discursive interactions (DI) orchestrated by the teacher educator (TE) provided clear evidence of learning among the pre-service teachers—most notably in PST4’s line-32 remark, “Right, I get it now! You left only pots on one side. That’s really interesting!”, which marks her moment of insight.

We also noted signs that Professional Learning Opportunities (PLOs) arose during the plenary. In line 25, for instance, TE built on PST1’s reflection about what they had learned from a peer (see line 13, where PST1 references PST3’s reasoning based on the principle of equivalence).

From these observations, two analytic categories emerged: (1) knowledge of content related to the properties of equality and (2) knowledge of mathematical practice associated with those properties.

The next section discusses these categories as they surfaced in the task resolutions and the accompanying DIs.

Category: knowledge of content related to the properties of equality

Most pre-service teachers did not apply the additive or multiplicative principles of equality. This omission may stem from their tendency to treat the equal sign merely as a “place to write the answer," rather than as a symbol of equivalence, a perspective that is essential for understanding equations (Trivilin & Ribeiro, 2015; Teles, 2002; Lessa, 2005).

Only two participants displayed knowledge consistent with algebraic thinking as characterized by Almeida (2016): establishing relationships, operating with unknowns, and constructing meaning. Their solutions evidenced these abilities through the use of multiple representational registers and a conceptual reading of both the task context and the equal sign as a symbol of equivalence (Fiorentini, Fernandes, & Cristóvão, 2005).

Category: knowledge of mathematical practice related to the properties of equality

All pre-service teachers solved the task using procedures likely acquired in their own schooling—chiefly, transposing terms from one side of the equation to the other, a technique still prevalent in many textbooks. However, most participants did not justify the procedures they employed, revealing limited articulation of the mathematical reasoning underlying their solutions.

Final Considerations

To address our research question—What mathematical and pedagogical knowledge do pre-service mathematics teachers possess and develop for teaching the properties of equality during a formative experience mediated by a Professional Learning Task (PLT)?—we designed a formative process, in collaboration with the instructor of Supervised Teaching Practicum III (ESO III), that followed the framework proposed by Ribeiro and Ponte (2020).

Using systematic comparisons within the MTSK framework, we analyzed the descriptive data gathered during the formative sessions. The analysis showed that the pre-service teachers exhibited professional knowledge in the subdomains KoT and KPM, which emerged as empirical categories from the first part of the PLT.

We also observed a marked reliance on the transposition technique, moving terms from one side of the equation to the other, rather than on the formal properties of equality. This tendency likely mirrors the way these future teachers learned mathematics in their own schooling.

Most participants still viewed algebra through a primarily “letter-based” lens (MEC, 1998), focusing on rules and procedures. A similar pattern was reported by Lautenschlager and Balvin (2021).

Our findings suggest that Professional Learning Tasks are promising tools: they surface pre-service teachers’ professional knowledge and spark mathematical and pedagogical discussion around the focal concept, thereby generating Professional Learning Opportunities (PLOs) (Ribeiro & Ponte, 2020).

We also affirm the value of the PLOT framework for structuring initial teacher-education activities.

Future studies should employ the PLOT model with varied configurations of PLTs and different mathematical topics; our own PLT addressed only the properties of equality.

We hope this study encourages further reflection and action on creating Professional Learning Opportunities from the outset of teacher preparation. Such efforts can tighten the links between theory and practice, and between university and school mathematics, ultimately strengthening professional knowledge and classroom practice in pursuit of high-quality mathematics education.

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